# Linear Analysis of a Force Reflective Teleoperator

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### Abstract

Complex force reflective teleoperation systems are often very difficult to analyze due to the large number of components and control loops involved. This document describes one model of a force reflective teleoperator and presents an analysis of the performance of the system based on a linear analysis of the general full order model. Reduced order models are derived and correlated with the full order models. Basic effects of force feedback and position feedback are examined and the effects of time delays between the master and slave are studied. The results show that with symmetrical position-position control of teleoperators, a basic trade off must be made between the intersystem stiffness of the teleoperator, and the impedance felt by the operator in free space.

### 1. Introduction

As man continues to expand into the extreme and dangerous environments of space and undersea, as well as having an increasing requirement to perform tasks in man-made hazardous environments such as nuclear reactors, it has become obvious that there is a need for systems which allow the manipulation of objects and effecting of the environment from a remote location. These systems range from mobile systems with limited ranges of motion, degrees of freedom and sensory capabilities to highly dextrous, force-reflecting, multi-arm and hand systems with a high degree of sensory capability. These more complex systems are often called teleoperation or telepresence systems. In unstructured environments in which the tasks to be performed are not known a priori, the manipulation of objects and the execution of complex tasks have shown themselves to be very formidable problems. While the concepts and basic theories involved in teleoperation systems have been investigated and studied for decades, there is still no highly dexterous, high performance system capable of reliably performing complex manipulation or assembly operations. And there are still many more barriers to overcome before reliable, robust, and effective systems are technologically and economically feasible.

# 2. The "Stick" Analogy for a Single-Degree-of-Freedom Teleoperator

What is the basic behavior we are trying to reproduce in the implementation of an actuated (active) force reflective teleoperation system? If we abstract the problem to that of the single-degree-of-freedom linear case, we are have simplified the problem to that depicted in Figure 1. This figure shows a simple "stick". This can be thought of as the most simple mechanical telemanipulator (1 DOF, linear, mechanically coupled). The behavior we are trying to reproduce when we separate the master and slave and attempt to actively couple them with actuation systems is the behavior of the single molecular layer  $(\delta x)$  between the two mechanically coupled systems. These desired behaviors include:

- low operator input impedance in free space
  - inertia
  - viscous drag
  - friction

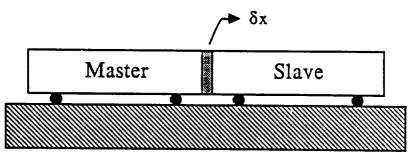


Figure 1: The simple passive stick teleoperator

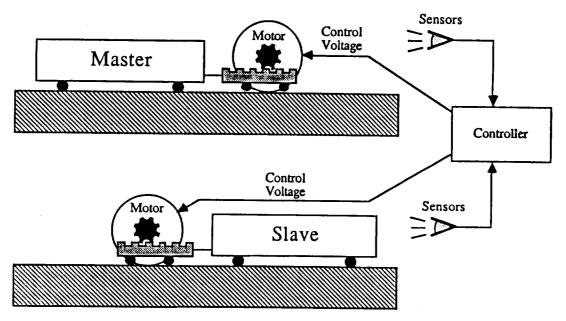


Figure 2: The active "stick" model of a simple one-degree-of-freedom teleoperator

- high intersystem stiffness
- high bandwidth force reflection
- stability for a wide range of contact impedances

These behaviors are some of the most difficult to actively implement in any effector system and reveal the severe demands placed on high performance teleoperation systems. These desired behaviors, however, can lend great insight into the performance and characteristics required of the individual components which comprise the system. This research will use these desired behaviors to determine the necessary characteristics which actuators, structures, sensors, and controllers must exhibit in order for the teleoperation system to perform as desired.

If the two ends of the "stick" are separated and an actuation system, controller, and sensors are integrated in each part, the model becomes that shown in Figure 2. Note that the basic subcomponents of the teleoperator are two symmetrical effector systems. In order to understand the behavior of such a system, we must first fully understand these effector subsystems which make up the teleoperator.

### 3. The Effector Model

Since a force reflecting teleoperator is really two effectors which have been connected via control systems to behave as a single system, the basic element which we must consider and evaluate in depth is the individual

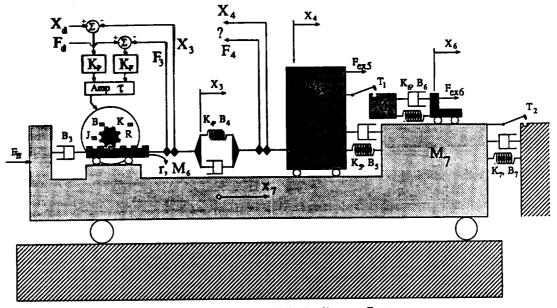


Figure 3: The general model of a linear effector

effector. The effector model we choose to analyze must be complex enough to exhibit what is observed in reality, yet it must be simple enough to allow intuitive understanding of the observed behaviors. The basic effector model which will be used in this study will be the model used in [6] and depicted in Figure 3. This model is presented as a fifth order system but reduced order second and third order simplifications are possible to describe system behavior in specific applications. The fifth, second, and third order models are derived from this general model by setting specific parameters as described in [6]. See [6] for a detailed description of how this may be accomplished.

This model can be configured as a bilateral teleoperation system by properly connecting the control systems and sensors. Then the effects of the various system components on overall teleoperator performance can be analyzed. Also, by using different control schemes in the connection of the two systems, the performance of the different control strategies can be rigorously studied and insight will be gained as to how a force reflecting teleoperator is to be properly controlled.

### 3.1 The Full Order Effector Model

By setting the parameters as shown in Table 1 [6], the simple fifth order effector model is generated. Two typical root locus plots for this configuration are shown in Figure 4. <sup>1</sup> Figure 4(A) shows a plot in which the force gain  $(K_f)$  is set to zero and the position gain  $(K_p)$  is varied from 0 to infinity. Figure 4(B) shows the same system with the position gain fixed at 10,000 volts/newton while the force gain is varied from 0 to infinity. Note that as the position gain is first increased, the poles which arise due to the interaction of the actuator and the structure migrate toward the imaginary axis becoming very much less damped. The lower magnitude poles represent an oscillation of the the load and the actuator. This interaction occurs at a lower frequency and while these poles do become less damped and increase in magnitude, they do not cross the imaginary axis, but approach the open loop zeros due to the damping within the structure. Note from Figure 4(B) that as the force gain is increased with a fixed position gain, the poles due to the actuator/load interaction become less damped and decrease in magnitude indicating a "softening" of the actuated system. The high frequency complex poles due to the actuator/structure interaction continue to move toward the imaginary axis, becoming less damped and eventually going unstable. The softening of the low frequency poles gives an indication of the backdriveability of the system. As these poles move toward the real axis, the system becomes more free and compliant and will more easily be "pushed around" by a disturbance. These

<sup>&</sup>lt;sup>1</sup>Note that the scale of the real and imaginary axes are markedly different in order to more easily see system behavior. Care should be exercised when attempting to extract exact frequency or damping ratio information from the plots. Note also that in many of the plots, the high frequency poles and zeros on the negative real axis are not shown on the plots since thier magnitudes are so great in comparison to the other frequencies of the system's components.

K <sub>m</sub>	51.9×10 <sup>3</sup>	N·m/a	Motor
R	.955	Ω	Motor
$J_m$	$38.1 \times 10^{-6}$	$kg \cdot m^2$	Motor <sup>†</sup>
$B_m$	$17 \times 10^{-6}$	$N \cdot m \cdot s/rad$	Motor
$\boldsymbol{L}$	0.188	mH	Motor
<b>T</b>	0	8	Amplifier
$\boldsymbol{G}$	108.7	$m^{-1} \rightarrow (1/r)$	Reducer
$B_3$	3.46	$N \cdot s/m$	Drive
$M_3$	0	kg	Drive
$B_4$	3.464	$N \cdot s/m$	Structure
$K_4$	$2.83 \times 10^{5}$	N/m	Structure
M <sub>5</sub>	2.65	kg	Load
$B_5$	17.32	$N \cdot s/m$	Load
$K_5$	113.2	N/m	Load
M <sub>6</sub>	0	kg	Touch Load
$B_6$	0	$N \cdot s/m$	Touch Load
$K_6$	0	N/m	Touch Load
$T_1$	0		Touch Load
$K_{\mathbf{p}}$	Variable	V/m	Controller
$\dot{K_f}$	Variable	$\dot{V/N}$	Controller
$K_d$	Variable	$V \cdot s/m$	Controller
Sf, Sp	Unity	·	Sensors
$M_7, B_7, K_7, T_2$	Fixed		Base

†Pitman Corporation Motor, Model 5113, winding #1 [1]

Table 1: The parameters used in the general model.

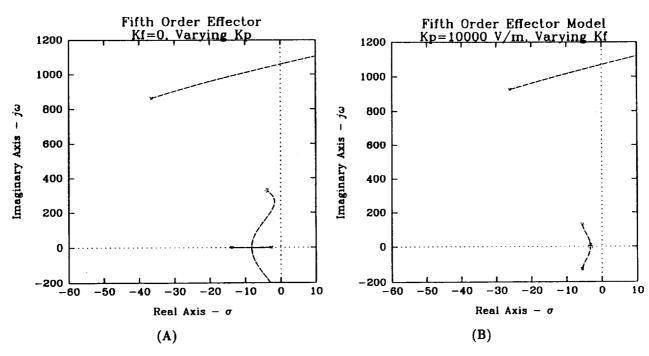


Figure 4: Root locus plots of the fifth order system. (A) shows the root locus with  $K_f$  set to 0, varying  $K_p$ . (B) shows the plot for  $K_p = 10000V/N$ , varying  $K_f$ .

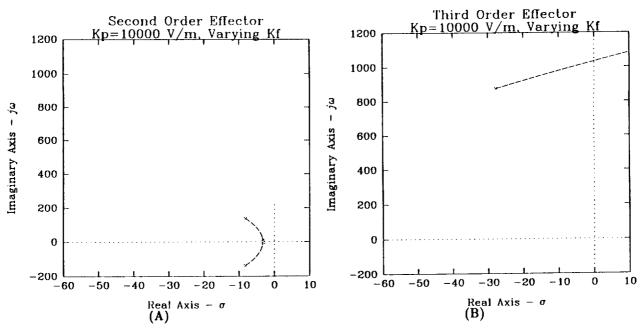


Figure 5: (A) is the root locus plot of the second order system varying  $K_f$ ,  $K_p = 10000 \text{ V/m}$ . (B) shows the same root locus for the third order system

poles approach the open loop zeros which would be the performance attained if the actuators became pure force sources with no acceleration or velocity dependent impedances.

### 3.2 Reduced Order Models

The fifth order effector model can be simplified for specific applications or to study specific interesting interactions. These reduced order models are described in detail in [6] and will only be reviewed for clarity here.

Rigid Structure with Moving Load and No Motor Inductance - If we assume that the structure is very rigid  $(K_4 \, large)$  and the motor inductance is zero (L=0), the model becomes a second order system used to examine interactions between the actuator and load. Figure 5(A) show a typical root locus for this system with  $K_f=0$ , varying  $K_p$ .

Compliant Structure with a Slow Load - If we assume that the load mass  $(M_5)$  is large and its motions are slow with respect to the dynamics of the actuator/structure interactions, the load mass can be assumed fixed and a third order model is generated which focuses on interactions of the actuator and structure. Figure 5(B) shows a typical root locus with  $K_p$  constant, varying  $K_f$ .

These simplified models show that we can use restricted generality models to understand specific behaviors of the full order system in particular situations. These lower order models allow a more intuitive understanding of the interactions among system elements and often make closed form solutions for particular performance criteria [6] possible.

## 4. The Teleoperator Model

### 4.1 The Tenth Order Teleoperator Model

By connecting two effector models, we can derive a model for a force reflecting teleoperation system. In this paper, we will restrict our analysis to a symmetrical position/position control law. In this method of control, the actual position of the master is used as the desired position for the slave(with no delay), and vice-versa. This control scheme leads to a tenth order model for the force reflective system. Figure 6 shows a simplified block diagram of the entire system.

Figure 7 shows two typical root locus plots corresponding to those in Figure 4. Note that the migration of the varying poles are very similar to those of the fifth order system, however the poles migrate more rapidly

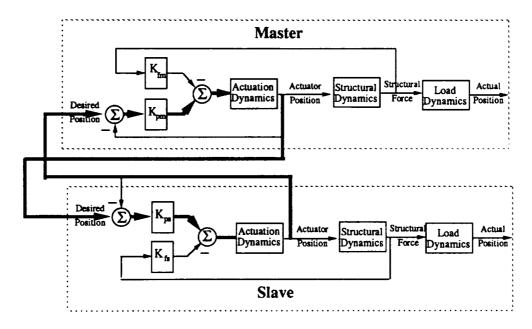


Figure 6: Block diagram of the tenth order teleoperation system composed of two fifth order effectors.

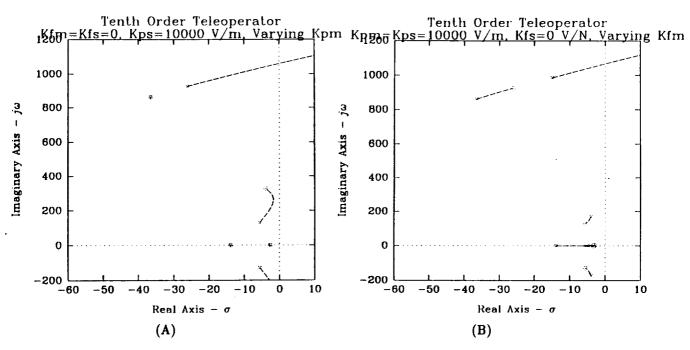


Figure 7: Typical root loci for the tenth order master/slave system derived by connecting two fifth order effector models. (A) shows the root locus for  $K_{f,master} = K_{f,slave} = 0$ ,  $K_{p,slave} = 10000 \text{ V/m}$ , varying  $K_{p,master}$ . This causes the intersystem stiffness to increase but at the same time changes the force reflection ratio (ratio of force applied by the slave to force applied by the operator) since only  $K_{p,master}$  varies with  $K_{p,slave}$  being help constant. (B) shows the rootlocus plot with  $K_{p,master} = K_{p,slave} = 10000 \text{ V/m}$ ,  $K_{f,slave} = 0 \text{ V/N}$ , varying  $K_{f,master}$ 

as the gains vary. Note also, that for each pole of the fifth order system varying  $K_p$ , there is a pole-zero-pole triad for the tenth order system, and only one of the poles of each triad actually migrates as the gains are varied. This pole-zero-pole coupling can be more easily understood by tracing the signal crossovers in Figure 6. The highlighted signal path shows that there is in actuality a positive feedback loop which starts at the desired position of the master, passes through the master's actuator, is fed to the desired position of the slave, and then through the actuator of the slave and back to the master as the master's desired position. This pathway is never inverted and therefore behaves like a positive feedback loop which causes the zeros to appear on the root locus plots in the same positions on the s-plane as the poles of the effector model. This positive feedback loop also causes some destabilization of the overall force reflective system.

In Figures 4(A) and 7(A), the poles due to the actuator/structure interactions cross the imaginary axis at a gain of 32805 volts/meter for the effector system but this value is reduced to 22806 volts/meter for the force reflective system. The sum of the position gains of the master and slave however is identical to the value for the effector. Thus, in some sense, these position gains add for the teleoperator. This indicates that it is much more difficult to achieve comparable position gains in a force reflective teleoperator than in a simple effector due the inherent nature of the required feedback necessary to connect two effectors into a force reflective teleoperator. Thus, given identical machinery, structures, sensors, and controllers, a force reflective teleoperator configured in a position-position mode will only be half as stiff (for a force reflection ratio of one) as an effector built out of the same components.

It is also interesting that the same mode goes unstable for both the effector and the teleoperator. The interaction between the actuator and structure is the first to cross the imaginary axis. While many teleoperators have well damped and stiff structural elements, it should be remembered that this compliance can also be thought of as a compliant drive element such as a transmission or drive cable/tendon. This high frequency "jitter" is often the limiting mode when the position or force gains are raised to too great a level.

In Figure 7(B), we see that as the force gain is increased while maintaining a constant position gain, the system becomes more backdriveable. A similar "softening" occurs as in the effector of Figure 4(B). In the same way that the effector becomes more free and backdriveable, the teleoperator becomes more free. This indicates a decrease in the amount of force required to move the system and will allow an operator to more easily position the teleoperator by decreasing the impedance of the system in free space. This will allow the operator to work more comfortably and for longer periods of time since the level of exertion required to move the system will be lowered. However, it must be remembered that since an increase in either the position gain or the force gain cause the high frequency poles to migrate toward the imaginary axis, the control engineer is faced with a tradeoff between intersystem stiffness and free space impedance.

#### 4.2 Reduced Order Model

If we wish to develop an intuitive understanding of the performance of teleoperators, we require a model which exhibits the behavior of the system, yet it must not be so complex as to not allow closed form analysis and the application of principles which are more easily applied to low order systems. However, even this limited complexity model of a teleoperator composed of two fifth order effectors has an order of ten. A model of this high an order does not generally allow closed form solutions for performance criteria and is difficult to understand without rigorous simulation and analysis. Even computer simulations can be painfully slow (and expensive). Therefore, a reduced order model which still embodies the behaviors of interest seems to be indicated.

For specific applications, many of the parameters included in the tenth order model become very small or very large or the eigenvalues associated with those parameters are no longer in a range of interest. Therefore, reduced order models are easily generated for some applications.

Third Order Master and Second Order Slave - If we wish to study the behavior of the system when the slave is in solid contact with a stiff object, we are able to reduce the order of the model to five. This is done by assuming that the master can be treated as the second order effector model and the slave can be thought of as the third order effector model presented in Section 3.2.

The master is attached to a human operator and interacts with his dynamics. In this case, one can assume that the structural compliance is small with respect to the compliance of the operator. We can also assume that the electrical time constant of the actuator is small with respect to the response of the operator which allows us to eliminate the motors inductance. This model is equivalent to the second order effector model described in Section 3.2.

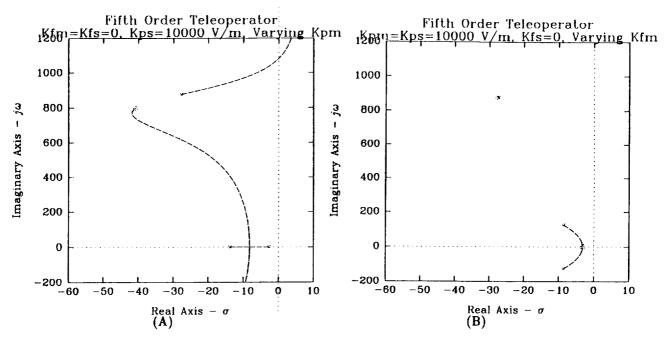


Figure 8: Root locus plots for the reduced order model a of force reflecting teleoperators. (A) shows the root locus for the fifth order master/slave system with  $K_{f,master} = K_{f,slave} = 0$ , varying  $K_{p,master}$ ,  $K_{p,slave} = 10000 V/m$ . (B) shows the root locus for the same system with  $K_{p,master} = K_{p,slave} = 10000 V/m$ ,  $K_{f,slave} = 0$ , varying  $K_{f,master}$ 

Since the slave is in contact with a rigid object, we can assume that the load mass is relatively unmovable and the structural or drive compliance is the dominating dynamic effect. This allows us to similarly use the third order model described in Section 3.2. These simplifications lead to a fifth order model which can be used to derive performance criteria important when the slave is in contact with stiff objects.

Figure 8(A) shows a plot of the root locus for this system as the position gain is varied with no force feedback. Note the similarities and differences between the full order model presented in Section 4.1 and this reduced model. Notice that the pole-zero-pole triads are no longer present since the models for the slave and the master are no longer identical. The agreement between the full order model and the reduced order model is very good for low gains, but as the gain is increased, the poles due to the actuator/load interactions of the master now approach the open loop zeros which occur due to the actuator/structure interactions of the slave rather than the open loop zeros of the master's actuator/structural damping/load interactions. These zeros no longer exist in the reduced order model of the master. The mode which goes unstable however is unchanged between the full order model and the reduced order.

Figure 8(A) shows the same system as we vary the force gain of the master, position gains being held constant. Observe again that the reduced order model depicts the same behavior as the tenth order system. As the force gain is increased, the system becomes more "free" and the dynamics of the master and slave begin to disappear. The system behaves as if the actuator is disconnected from the system at the master, but the forces applied by the slave are still felt by the operator. The system becomes more like the ideal stick model described in Section 2.

## 5. The Effect of Time Delays Between Master and Slave

Often, in real teleoperation systems, there is a delay between the master and slave. This may be due to transmission delay as in space applications, where the delay may be as great as a few seconds, or it may be due to computational delay if a digital control system is interposed between the two subsystems. In this case the delay may be small, such as a few milliseconds. In our experience at the Center for Engineering Design, we have discovered that even a small delay between the master and slave can cause a serious degradation of overall system performance, especially in the areas of intersystem stiffness and free space operator input impedance. We can study the effects of this delay by interposing a first order lag between the master and

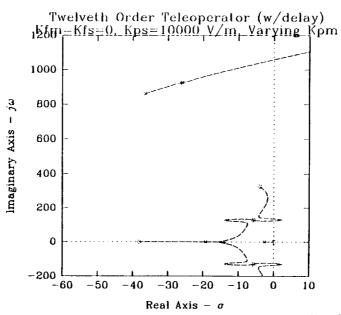


Figure 9: Root locus for the twelveth order system derived from two fifth order effectors with transmission delays between the master and slave and between the slave and master,  $K_{f,master} = K_{f,slave} = 0$ , varying  $K_{p,master}$ ,  $K_{p,slave} = 10000 V/m$ . (B) show the locus for  $K_{p,master} = K_{p,slave} = ???$  V/m, varying  $K_f$ 

slave and between the slave and the master. We can then compare the results of the root locus analysis from the models without the delay to those with the delay. This will allow us to make inferences as to how the delay will effect overall system performance.

If we interpose the delays in the intersystem connections of the tenth order model, each delay adds another order for a total of twelve. Figure 9 shows root a locus plot for this twelveth order system varying  $K_p$  with  $K_f = 0$ .

Observe that the pole-zero-pole triads are no longer present. The delay causes the poles and zeros to separate and therefore the pole migrates to the shifted zero. More importantly, however, notice that in Figure 9(A), some poles due to the actuator/load interactions have been shifted to the left by the delay and move out the negative real axis to zeros arising due to the delays. This implies that the system has a higher natural frequency and is more damped for equivalent gains, in comparison to the teleoperator without the delays. The poles due to the actuator/structure interactions however, remain relatively unchanged. This means that the high frequency "jitter" is unchanged by the delay, but the lower frequency poles are more damped and less "free". Thus, the slew drag is increased by the delay and increasing the appropriate gains to minimize the effect is impossible. Notice also, that the actuator/load interactions cross the imaginary axis at a fairly low frequency (125 radians/sec.) and at a relatively low gain (8084 volts/meter).

### 6. Conclusions

A general model of a force reflecting teleoperation system was derived in order to examine some basic effects of position and force feedback and the inherent tradeoffs between intersystem stiffness and free space tel which must be made when setting these gains. A reduced order model was generated for specific situations of the master and slave in order to more easily understand observed behaviors. The root locus analysis applied shows that there are intrinsic trade offs which are made as we increase either the position gain or the force gain. The control engineer must balance the intersystem stiffness of the system against the impedance felt by the operator as he moves the system in free space. The effects of delays between the master and slave on the achievable intersystem stiffness and slew drag were examined. This shows that even small delays degrade the performance of the system by causing the actuator/load interactions to become more damped and sluggish. This can either be tolerated or can be minimized by increasing the force gain. If the force gain is increased however, the position gain must simultaneously be decreased, thereby degrading intersystem stiffness.

In the future, additional reduced order models will be derived and used to find closed form and numerical

Quantitaive Performance Criteria (QPC's) [6]. These performance criteria will enable a designer to easily see the impact of design decisions on the overall performance of the system. This should help a designer of a teleoperation system to more easily balance the conflicting constraints to satisfactorily meet the design goals.

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